Admission Test (with solutions ✓)

MOODLE Instructions :

- You have 90 minutes do complete the test.
- There are 20 questions.
- THE TEST IS DONE IN SEQUENTIAL MODE: YOU MUST SUBMIT THE ANSWER OF THE QUESTION BEFORE MOVING TO THE NEXT QUESTION.
- ONCE SUBMITTED YOU MAY NOT REVIEW YOUR ANSWER.
- After submitting the test you have 15 minutes to upload a file with the handwritten answers and all the calculus that you performed to solve the question.
- A correct choice is 1 point. A wrong choice is -0.05.

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1. The value of $(A \vee (B \wedge C))$ and $((A \vee B) \Rightarrow C)$ for A=T (True), B=T (True) and C=F (False) is respectively:

   (a) F and F

   (b) F and T

   (c) T and F ✓

2. Consider the following logical propositions

$$a : \forall x \in \mathbb{Z},\ \exists y \in \mathbb{N}_0 : x^2 = y$$

   The $\neg a$ and logical value of a are:

   (a) $a = T$ and $\neg a = \forall x \in \mathbb{Z},\ \exists y \in \mathbb{N} : x^2 \neq y$

(b) $a = T$ and $\neg a = \exists x \in \mathbb{Z},\ \forall y \in \mathbb{N} : x^2 \neq y$ ✓

(c) $a = F$ and $\neg a = \exists x \in \mathbb{Z},\ \forall y \in \mathbb{N} : x^2 \neq y$

3. Let $A = \{x \in \mathbb{R} \ : |x-1| > 2\}$ and $B = \left\{x \in \mathbb{R} \ : \frac{x^2}{3} \leq 3\right\}$.
   $A \cap B$ and $B \setminus A$ are:

   (a) $A \cap B = [-3, -1]$ and $B \setminus A = [-1, 3[$

   (b) $A \cap B = \emptyset$ and $B \setminus A = [0, 3]$

   (c) $A \cap B = [-3, -1[$ and $B \setminus A = [-1, 3]$ ✓

4. Simplifying for all $a, b \in \mathbb{R}^+$

$$\frac{c(2ab^{-2})^2}{c^{-3}\frac{ab^3}{b^2}}$$

   we obtain:

   (a) $\frac{2c^4a}{b^{-3}}$

   (b) $\frac{4c^4a}{b^5}$ ✓

   (c) $\frac{2c^4a^3}{b}$

5. The polynomial $h(x)$ with roots $x = 1$, $x = 2$ and $x = 0$ and such that $h(-1) = -12$ is

   (a) $h(x) = 2x^3 - 6x^2 + 4x$ ✓

   (b) $h(x) = x^3 - 3x^2 + 2x$

   (c) $h(x) = 2x^3 - 4x^2 - 6$

6. The geometric progression u_n with $u_1 = -0.5$ and with common ratio 2 is

   (a) monotonically increasing

   (b) monotonically decreasing ✓

   (c) not monotone

7. The term of order 5 of the sequence $v_n = \dfrac{(-1)^n 5n}{n+5}$ is

   (a) $\dfrac{25}{10}$
   (b) $\dfrac{-5}{2}$ ✓
   (c) $\dfrac{-25}{2}$

8. The limit of the following sequence ($n \in \mathbb{N}$)

$$\lim_{n \longrightarrow +\infty} \left(\frac{2n^3 + 3n}{2n\sqrt{n^4+1}} \right)$$

   is:

   (a) $+\infty$
   (b) 0
   (c) 1 ✓

9. The limit of the following sequence ($n \in \mathbb{N}$)

$$\lim_{n \longrightarrow +\infty} \left(\frac{2n+1}{2n+2} \right)^n$$

   is

   (a) 1
   (b) e^2
   (c) $\frac{1}{\sqrt{e}}$ ✓

10. The solution set in $\mathbb{R}$ of the inequality: $(x^3 - 3x)(e^x - 1) \geq 0$ is

   (a) $]-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty[$
   (b) $]-\infty, -\sqrt{3}] \cup \{0\} \cup [\sqrt{3}, +\infty[$ ✓
   (c) $[-\sqrt{3}, \sqrt{3}]$

11. The set of real numbers that is the solution of he inequality

$$\log_3(1-x) \leq 1$$

is

(a) $[-2, 1[$ ✓
(b) $[-1, 2[$
(c) $]-\infty, -2]$

12. Consider function $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined by the expression

$$f(x) = \log(x-3)$$

The domain of f is:

(a) $\mathbb{R}$
(b) $]0, +\infty[$
(c) $]3, +\infty[$✓

13. Consider the real function of real variable defined by,

$$f(x) = \begin{cases} \sqrt{\dfrac{e^x - 1}{x}} & \text{for } x \geq 0 \\ \dfrac{2x+1}{x^2-1} & \text{for } -\infty < x < 0 \end{cases},$$

The domain of f is,

(a) $]-\infty, 0[\cup]0, +\infty[$.
(b) $\mathbb{R} \setminus \{-1, 0\}$. ✓
(c) $]-\infty, -1[\cup]-1, +\infty[$

14. The inverse function of $h(x) = e^{x+1}$ is

(a) $g(x) = \ln(x-1)$
(b) $g(x) = \ln(x) - 1$✓
(c) $g(x) = e^{x-1}$

15. Consider the real function of real variable defined by,

$$g(x) = \begin{cases} x^2+1 & \text{for } x > 0 \\ 2 & \text{for } x = 0 \\ 2x+1 & \text{for } -\infty < x < 0 \end{cases},$$

(a) g is not defined in $x = 0$.

(b) g is continuous in $x = 0$

(c) $\lim_{x \to 0^+} g(x) = 1$ ✓

16. Consider the real function of real variable defined by,

$$h(x) = \begin{cases} x^3+2x & \text{for } x \geq 0 \\ 2x^2+x & \text{for } -\infty < x < 0 \end{cases},$$

(a) h is differentiable at $x = 0$ and $h'(0) = 2$.

(b) The left derivative of h at $x = 0$, $h'_l(0) = 2$.

(c) The right derivative of h at $x = 0$, $h'_r(0) = 2$. ✓

17. The derivative function of $h(x) = \dfrac{e^x - 1}{e^x + 1}$ is:

(a) $h'(x) = \dfrac{2e^x}{(e^x+1)^2}$ ✓

(b) $h'(x) = \dfrac{e^{2x}+e^x}{(e^x+1)^2}$

(c) $h'(x) = \dfrac{e^x}{(e^x+1)}$

18. The derivative function of $t(x) = (3x^2 + x)^2$ is:

(a) $2(6x+1)$

(b) $4x(3x^2+x)$

(c) $12x(3x^2+x)$ ✓

19. Consider the function $f(x) = e^{x^2+k}$ where k is a constant real number. Knowing that $f'(1) = 2$ then k must be equal to

(a) $\log 2 - 1$

(b) -1 ✓

(c) 0

20. The point(s) of the graphical representation of the function $h(x) = \frac{1}{x}$ at which the tangent line is parallel to the line $y = -x + 2$ is(are):

(a) (-1,1)

(b) (-1,-1) and (1,1) ✓

(c) (1,1)