



MARIE SKŁODOWSKA-CURIE POSTDOCTORAL FELLOWSHIPS 2025 EXPRESSION OF INTEREST FOR HOSTING MARIE CURIE FELLOWS

HOST INSTITUTION

NOVA University Lisbon

RESEARCH GROUP AND URL

Analysis Group of the Center for Mathematics and its Applications (NOVA Math) <u>https://novamath.fct.unl.pt/</u> https://novamath.fct.unl.pt/research-groups/analysis

SUPERVISOR (NAME AND E-MAIL)

Oleksiy Karlovych oyk@fct.unl.pt

SHORT CV OF THE SUPERVISOR

Oleksiy Karlovych graduated with honour from the Odesa State University (Ukraine) in 1995 with the MSc degree in Mathematics and got his PhD in Mathematical Analysis from the Rostov-on-Don State University (Russian Federation) in 1998. He started his career as an Assistant Professor in theSouth-Ukrainian State Pedagogical University in Odesa (Ukraine). In 2000, Karlovych moved to Portugal. In 2000-2004 and 2006-2007 he was a post-doctoral research fellow in Instituto Superior Técnico, Lisbon. In 2004-2006 he was an Invited Assistant Professor in University of Minho in Braga (Portugal). In 2008 he joined the Mathematics Department of the NOVA School of Science and Technology of the NOVA University Lisbon as an Assistant Professor. He was promoted to an Associate Professor in 2012. In 2014-2018 Karlovych was a Coordinator of the PhD Program in Mathematics. Since 2014 he is a Coordinator of the Analysis Group of the Center for Mathematics and Applications (NOVA Math). In 2014-2022 he was a member of the Council of the Mathematics Department.

Karlovych's research interests lie in the study of Toeplitz, singular integral, and convolution-type operators on Banach function spaces, including Orlicz, Lorentz, rearrangement-invariant, and variable Lebesgue spaces.

Karlovych published more than 75 papers in journals, including Journal of Functional Analysis, Integral Equations and Operator Theory, Proceedings of the American Mathematical society, Proceedings of the London Mathematical Society, Journal of Mathematical Analysis and Applications. He also published about 30 papers in book chapters.

Karlovych co-supervised 5 MSc theses. Currently he supervises 3 PhD theses.

He was a co-organizer of several conferences in Operator Theory, including the International Workshop on Operator theory and Applications, IWOTA 2019 in Lisbon with more than 450 participants. He was a co-editor of 4 books published in the prestigious series Operator Theory: Advances and Applications, Springer.

For more details, see <u>https://www.cienciavitae.pt//en/C913-2961-97DC</u>

5 SELECTED PUBLICATIONS

O. Karlovych, E. Shargorodsky, When are the norms of the Riesz projection and the backward shift operator equal to one? Journal of Functional Analysis. 285.12 (2023): Paper No. 110158. https://www.sciencedirect.com/science/article/pii/S0022123623003154





G. Curbera, O. Karlovych, E. Shargorodsky, On the full range of Zippin and inclusion indices of rearrangement-invariant spaces. Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas. 118 (2024): 93. https://link.springer.com/content/pdf/10.1007/s13398-024-01599-8.pdf

C. Fernandes, O. Karlovych, M. Valente., On the density of Laguerre functions in some Banach function spaces. Journal of Inequalities and Special Functions. 13.2 (2022): 37-45. http://www.ilirias.com/jiasf/repository/docs/JIASF13-2-4.pdf

O. Karlovych, S. Theampi, The Brown-Halmos theorem for discrete Wiener-Hopf operators, Advances in Operator Theory. 9 (2024): 69.

https://link.springer.com/content/pdf/10.1007/s43036-024-00370-5.pdf

O. Karlovych, E. Shargorodsky, The Coburn lemma and the Hartman-Wintner-Simonenko theorem for Toeplitz operators on abstract Hardy spaces. Integral Equations and Operator Theory. 95 (2023): 6. https://link.springer.com/article/10.1007/s00020-023-02725-8

PROJECT TITLE AND SHORT DESCRIPTION

Toeplitz operators on abstract Hardy spaces

The theory of Toeplitz operators is a vital part of modern analysis, with applications to moment problems, orthogonal polynomials, approximation theory, integral equations, bounded- and vanishing-mean oscillations, and asymptotic methods for large structured determinants, among others. This area of analysis includes Wiener's filtering problems, the statistical physics of gases, diverse moment problems, ergodic properties of random processes, complex interpolation, etc. Toeplitz operators constitute a very interesting class of non-selfadjoint operators on Hardy-type spaces with very rich spectral theory and whose study uses deep connections between functional analysis, operator theory, complex analysis, and general topology.

A bounded linear A operator on a Banach space is said to be Fredholm if the dimensions of its kernel and its cokernel are finite. In the latter case, the difference of these dimensions is called the Fredholm index of A denoted by Ind A. This project is devoted to the study of Fredholm properties of Toeplitz operators in the nonclassical setting. Let X be a Banach function space on the unit circle T (see [BS88]) and H[X] be the respective abstract Hardy space, that is, the subspace of X consisting of functions, whose Fourier coefficients with negative indices vanish. The canonical projection from the space X onto its subspace H[X] is usually called the Riesz projection and is denoted by P. We will always suppose that the Riesz projection P from X onto H[X] is bounded. For an essentially bounded complex-valued function a on the unit circle T, the Toeplitz operator with symbol a is defined by T(a)f=P(af). We are interested in a characterisation of the Fredholm property and an index formula for the operator T(a) in terms of its symbol a.

The spectral theory of Toeplitz operators is completely understood in the case when X is the reflexive Lebesgue space Lp and the symbol a is a continuous or piecewise continuous function. Our focus in this project will be on more general Banach function spaces X, which are not necessarily reflexive, that is, when the second Banach dual space X^{**} is larger than the space X. The most natural example of Banach function spaces we are going to consider is the scale of Lorentz spaces Lpq with p strictly between one and infinity. This scale extends the scale of Lebesgue spaces Lp. For extreme values of q, the space Lpq is not reflexive, and it is even nonseparable when q is equal to infinity. The Lorentz space Lp,infinity (the weak Lp-space) is a very important object of study that appears in many branches of modern functional analysis, harmonic analysis, and operator theory. Very little is known on spectral properties of Toeplitz operators on this space. The main aim of this project is to extend the existing Fredholm theory of Toeplitz operators with piecewise continuous symbols on abstract Hardy spaces built upon reflexive Banach function spaces X to the nonreflexive setting. We are going to give a complete description of Fredholm





Toeplitz operators T(a) with piecewise continuous symbols a on all Hardy-Lorentz spaces H[Lpq] for all values of p and q such that the Riesz projection is bounded from Lpq onto H[Lpq].

Typically, for reflexive Banach function spaces, duality arguments are frequently used in the proofs. Unfortunately, such arguments are not available in general, when the reflexivity is lost. This fundamental obstacle requires new methods for the study of spectral properties of Toeplitz operators in our setting. We believe that such methods, developed in the course of our work, might be also useful for the study of other classes of operators on Banach function spaces.

SCIENTIFIC AREA WHERE THE PROJECT FITS BEST*

Mathematics (MAT)